LYCEN 9843

CHIRAL SYMMETRY RESTORATION

AND PION INTERACTION IN NUCLEAR MATTER

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Abstract

This paper is devoted to the interplay between p-wave, s-wave pion-nucleon/nucleus interaction and in-medium pion-pion interaction with special emphasis on the role of the nuclear pionic scalar density driving a large amount of chiral symmetry restoration. In particular we show that the πNN *coupling constant and the Goldberger-Treiman relation are preserved in the nuclear medium within certain conditions. We also discuss the related problem of the in-medium pion-pion strength function.*

1 Introduction

The collective pionic modes originating mainly from the coupling of the pion to delta-hole configuration have suscitated considerable interest. As proposed in [1] and confirmed by detailed calculations ([2, 3]), the direct experimental evidence of their existence even at peripheral nuclear density is provided by charge exchange reactions [4, 5]. These collective modes, sometimes called pisobars, are expected to play a prominent role in highly excited hadronuclear matter produced in relativistic heavy ion collisions. Indeed, it has been predicted [6, 7, 8] that they may significantly affect the rho meson mass distribution at large density. As shown in [9, 10], they are able to explain an important part of the excess of dilepton production rate in the 500 MeV invariant mass region observed by the CERES [11] and HELIOS [12] collaboration at CERN/SPS. In addition it has also been predicted [13] an important reshaping of the pion-pion interaction in the scalar-isoscalar channel ("sigma" channel), producing a sizeable accumulation of strength near or even below two-pion threshold. A detailed discussion of this problem, has been given in [14]. In particular, the influence of chiral symmetry constraints, stopping the subthreshold invasion of strengh due to the coupling of the pion to nucleon-hole states, have been extensively investigated. It has been soon realized that this medium effect is of considerable importance for the still open problem of nuclear saturation since an important part of the nucleon-nucleon attraction comes from correlated two-pion exchange and several papers have brought extremely interesting results [15, 16, 17]. However, this in-medium modification of the NN interaction is obviously very sensitive to the precise low mass shape of the in-medium $\pi\pi$ strength function. In that respect Oset *et al.* have drawn the attention on vertex correction on in-medium pion-pion interaction imposed by the standard chiral lagrangian [18]. According to this paper some strength should appear below the two-pion threshold at densities below normal nuclear matter density.

One of the common features of the problems mentionned above, is the interplay between p-wave pion-nucleon/nucleus interaction, s-wave pion-nucleon/nucleus interaction and in-medium

pion-pion-interaction all being related to the amount of chiral symmetry restoration in the nuclear medium. The aim of this paper is to clarify this important question in a unified scheme based on the standard chiral lagrangian treating simultaneously pion propagation, pion-pion interaction and chiral symmetry restoration in the nuclear medium. In section 2, we show how the pion scalar density $< \Phi^2 >$ (directly related to the pion cloud contribution of the pion-nucleus sigma term governing chiral symmetry restoration) may *a priori* renormalize the πNN coupling in the nuclear medium. Starting from the in-medium modified pion-pion interaction, we discuss in section 3 how this scalar density governs the s-wave $\pi-nucleus$ interaction through the interaction of the external pion with the nuclear pion cloud; we also give an explicit expression (which slightly differs from [19]) of the pion-nucleus sigma commutator in terms of the full longitudinal spin-isospin response function. We discuss the consequences on the pion propagator in section 4, showing in particular that the p-wave πNN coupling constant is actually unmodified in the medium to one pion loop order. Finally we demonstrate in section 5 that the presence of the $3\pi NN$ interaction cannot modify the pion-pion strength function at finite density at variance with the conclusion of [18].

2 The chiral lagrangian

All the results will be derived within a chiral $SU(2)\times SU(2)$ non linear lagrangian with anexplicit chiral symmetry breaking piece containing a nucleon field N and a isovector pion field $\vec{\Phi}$ embedded in the 2×2 matrix :

$$U=\xi^2=e^{i\vec{\tau}.\hat{\Phi}F(X)}, \qquad X^2=\frac{\Phi^2}{f_\pi^2}, \qquad F(X)=X+\alpha X^3+\alpha' X^5+... \tag{1}$$

where $F(X)$, which is an odd function of X, defines the particular non linear representation. The most usual choices are the PCAC choice ($F=\arcsin X$, $\alpha=1/6$), the Weinberg choice ($\alpha=-1/12$) and the choice $F=X$ commonly used in chiral pertubation expansion. Since the physical observables should not depend on the representation we will not specify it as a consistency check of the results. In the following we will treat the soft pion interaction to one pion loop order and a practical consistency test will be the independence of our results with respect to the parameter α. The lagrangian is :

$$\mathcal{L}=\frac{f_\pi^2}{4}\,tr\partial_\mu U\partial^\mu U^\dagger+\frac{1}{4}f_\pi^2 m_\pi^2\,tr(U+U^\dagger)+i\bar{N}\gamma^\mu\partial_\mu N-M_N\bar{N}N+\bar{N}\gamma_\mu\mathcal{V}_c^\mu N+g_A\,\bar{N}\gamma_\mu\gamma^5\mathcal{A}_c^\mu N \tag{2}$$

with

$$\mathcal{V}_c^\mu=\frac{i}{2}\left(\xi\partial_\mu\xi^\dagger+\xi^\dagger\partial_\mu\xi\right) \qquad \mathcal{A}_c^\mu=\frac{i}{2}\left(\xi\partial_\mu\xi^\dagger-\xi^\dagger\partial_\mu\xi\right) \tag{3}$$

To lowest order the pion-pion interacting lagrangian is :

$$\mathcal{L}_{4\pi}=\frac{1}{f_\pi^2}\left[-m_\pi^2\left(\alpha-\frac{1}{24}\right)\Phi^2\,\Phi^2\,+\,\left(\alpha-\frac{1}{6}\right)\Phi^2\,\partial_\mu\vec{\Phi}.\partial^\mu\vec{\Phi}\,+\,\left(2\alpha+\frac{1}{6}\right)\vec{\Phi}.\partial_\mu\vec{\Phi}\,\vec{\Phi}.\partial^\mu\vec{\Phi}\right] \tag{4}$$

The πNN and the $3\pi NN$ pieces are :

$$\mathcal{L}_{\pi NN}=\frac{g_A}{f_\pi}\,\bar{N}\gamma^\mu\gamma^5\frac{\vec{\tau}}{2}.\partial_\mu\vec{\Phi}N \tag{5}$$

$$\mathcal{L}_{3\pi NN}=\frac{g_A}{f_\pi^3}\,\bar{N}\gamma^\mu\gamma^5\frac{\vec{\tau}}{2}.\left[\left(\alpha-\frac{1}{6}\right)\Phi^2\partial_\mu\vec{\Phi}+\left(2\alpha+\frac{1}{6}\right)\vec{\Phi}\,\vec{\Phi}.\partial_\mu\vec{\Phi}\right]N \tag{6}$$

It follows that, to one pion loop order, the in-medium effective πNN lagrangian becomes :

$$\mathcal{L}^{eff}_{\pi NN} = \frac{g_A}{f_\pi}\left(1+\beta\frac{<\Phi^2>}{6f_\pi^2}\right)\bar{N}\gamma^\mu\gamma^5\frac{\vec{\tau}}{2}.\partial_\mu\vec{\Phi}N, \qquad \beta = 1+10\left(\alpha-\frac{1}{6}\right) \tag{7}$$

In principle the expectation value of Φ^2 renormalizing a nucleon vertex does not correspond to its averaged value due to the presence of short range correlations. This effect has been discussed in a recent paper [20] and found to be moderate because the scalar pion cloud of a given nucleon is mainly outside of the correlation hole. In the following, we disregard this effect of short range correlation and calculate the pion loop $<\Phi^2>$ with the in-medium pion propagator including the effect of the p-wave interaction from $\mathcal{L}_{\pi NN}$. The retarded pion propagator D_R and the time ordered pion propagator D_R^{++} have the form :

$$\begin{aligned} D_R(q) &= \left(q_0^2-\omega_q^2-\mathbf{q}^2\tilde{\Pi}^0(q_0,\mathbf{q})\right)^{-1} \\ D_R^{++}(q) &= (1+f(q_0))D_R(q)-f(q_0)D_R^*(q) \end{aligned} \tag{8}$$

from which we get :

$$<\Phi^2>= 3\int\frac{i\,d^4k}{(2\pi)^4}\,e^{ik_0\eta_+}\,D_R^{++}(k) \;= 3\int\frac{d^3k}{(2\pi)^3}\int_0^\infty dE\left(-\frac{2E}{\pi}\,ImD_R(E,\mathbf{k})\right)\frac{1+2f(E)}{2E} \tag{9}$$

where the second form incorporating the thermal factor ($f(E)=(exp(\beta E)-1)^{-1}$) can be used at finite temperature if needed. In the previous expressions the contribution of the free space propagator is understood to be always substracted. In the following we will work most of the time at zero temperature and make no distinction between time ordered and retarded propagators which coincide for positive frequencies. All the self-energies and related quantities will be expressed in terms of time ordered propagators as usual. It is nevertheless interesting to remark that most of the formal results will be valid at finite temperature by replacing the energy integral by Matsubara frequencies summation, propagators by Matsubara propagators and finally taking the analytical continuation ($i\omega_n \rightarrow \omega+i\eta$) to obtain the corresponding self-energy. With this lagrangian, the pion polarizability is $\tilde{\Pi}^0=(g_A/2f_\pi)^2U_N$ where U_N is nothing but the standard nucleonic Lindhardt function (with an explicit factor 4 for spin-isospin). However to make contact with the well established phenomenology of nuclear collective pionic modes, one can incorporate as usual the effect of the delta as well as the screening effect from short range correlations (g' parameters); details can be found in many papers, see e.g. [6, 14].

3 s-wave pion-nucleus interaction and pion-nucleus sigma commutator

In free space, the $\pi\pi$ interaction potential in the $I=0$ channel can be straightforwardly obtained from $\mathcal{L}_{4\pi}$:

$$<q_aq_b|\mathcal{M}_0|q_cq_d>=\frac{1}{f_\pi^2}\left(m_\pi^2-2s\,+\,\beta\sum_{i=1}^{4}(m_\pi^2-q_i^2)\right), \qquad \beta=1+10\left(\alpha-\frac{1}{6}\right) \tag{10}$$

where $s=(q_a+q_b)^2=(q_c+q_d)^2$ is the center of mass energy of the pion pair. For on-shell pions ($q_i^2-m_\pi^2=0$) the α dependent piece proportional to the inverse pion propagators disappears as

it should be. When two pions are soft (i.e. $q_a = q_c = 0$), in the PCAC representation ($\alpha = 1/6$), this amplitude satisfies the soft pion theorem

$$\mathcal{M}_0\,(s = u = m_\pi^2, t = 0) \; = \frac{m_\pi^2}{f_\pi^2} = \frac{\Sigma_{\pi\pi}}{f_\pi^2} \tag{11}$$

where $\Sigma_{\pi\pi} = m_\pi^2$ is the pion-pion sigma commutator (with non relativistic normalization for pion states its value is $m_\pi/2$). This result has to be compared with the Weinberg results for on-shell pions at threshold $\mathcal{M}_0\,(s = 4m_\pi^2, u = t = 0) = -7m_\pi^2/f_\pi^2$. The very important practical consequence of these chiral constraint is that the potential, although attractive in the threshold region, has to become repulsive somewhere below, thus preventing the invasion of strength and s-wave pion pair condensation in the nuclear medium [14].

In the medium this pion-pion potential receives vertex corrections depicted on fig.1, which can be calculated from $\mathcal{L}_{\pi NN}$ and $\mathcal{L}_{3\pi NN}$. For a zero momentum pion pair ($\mathbf{P} = 0$), the effective $I = 0$ in-medium pion-pion potential takes the very simple form :

$$< q_a q_b|\mathcal{M}_0^{eff}|q_c q_d > = \frac{1}{f_\pi^2}\left(m_\pi^2 - 2s \,+\, \beta\sum_{i=1}^{4}\left(m_\pi^2 - q_i^2 + \mathbf{q}_i^2\,\tilde{\Pi}^0(\omega_i, \mathbf{q}_i)\right)\right) \tag{12}$$

Hence, the effect of the vertex corrections depending on the p-wave pion polarizabilities at each pion leg is to make the quasi-potential representation independent (*i.e.* α independent) for on-shell quasi-pions satisfying $q_i^2 - m_\pi^2 - \mathbf{q}_i^2\,\tilde{\Pi}^0(\omega_i, \mathbf{q}_i) = 0$.

The s-wave isoscalar pion scattering amplitude in the medium receives a contribution from the interaction of the external pions with the pion cloud. Using crossing symmetry, this amplitude can be expressed in terms of the previous effective interaction :

$$\begin{aligned} < q_1|T|q_2 > \;&=\; T(q_1^2, q_2^2, t) = \frac{V}{2}\int \frac{i d^4k}{(2\pi)^4}\, D_R(k)\; < q_1, -q_2|\mathcal{M}_0^{eff}|k, -k > \\ &=\; \frac{V}{2f_\pi^2}\int \frac{i d^4k}{(2\pi)^4}\, D_R(k)\, \left(m_\pi^2 - 2t - \beta\,(2D_R^{-1}(k) + q_1^2 - m_\pi^2 + q_2^2 - m_\pi^2)\right) \end{aligned} \tag{13}$$

where t is the Mandelstam variable $t = (q_1 - q_2)^2$ and V the volume of the hadronic matter. The contribution of the inverse pion propagators disappears once the vacuum contribution is substracted. Let us consider a piece of nuclear matter with A nucleons and density $\rho = A/V$; the pion cloud contribution to the pion-nucleus sigma term per nucleon is [19] :

$$\Sigma_A^{(\pi)} = \frac{m_\pi^2}{2\rho} < \Phi^2 > +\mathcal{O}(< \Phi^4 >) = \frac{3m_\pi^2}{2\rho}\int \frac{i d^4k}{(2\pi)^4}\, D_R(k) \tag{14}$$

Hence, in the PCAC scheme ($\beta = 1$), this pion amplitude (per nucleon) takes the form :

$$\frac{1}{A}T(q_1^2, q_2^2, t) = \frac{\Sigma_A^{(\pi)}}{f_\pi^2 m_\pi^2}\left(m_\pi^2 - t + \frac{1}{3}(t - q_1^2 - q_2^2)\right) \tag{15}$$

and satisfies the well known PCAC constraints for $\nu_B = (t - q_1^2 - q_2^2)/4M_N = 0$; namely : soft point : $T(0,0,0) = \Sigma_A/f_\pi^2$; Cheng-Dashen point : $T(m_\pi^2, m_\pi^2, 2m_\pi^2) = -\Sigma_A/f_\pi^2$; Adler consistency relation : $T(m_\pi^2, 0, m_\pi^2) = T(0, m_\pi^2, m_\pi^2) = 0$. One may notice that the amplitude at threshold is one third of the soft pion point. In particular, in the nucleon case, this gives an isospin symetric scattering length $a_0 = -\Sigma_N^\pi/12\pi f_\pi^2$; taking for the pion cloud contribution to the pion-nucleon

sigma commutator a value of about 30 MeV (see [21, 22] and discussion below) one obtains $a_0 \approx -0.012 m_\pi^{-1}$ which is rather close to the experimental value. However this agreement is probably accidental since many other effects may contibute to the scattering length. In first rank the Born term alone also gives approximatively the experimental value; the valence quark contribution to the sigma term, the delta contribution should be also incorporated; similarly in the nuclear case, effect of rescattering of the isovector amplitude in presence of Pauli correlation are known to be important. Since s-wave scattering length is not the purpose of this paper we do not elaborate on it, although our approach might be implemented in this direction along the lines of [23].

The important quantity to be evaluated is $< \Phi^2 > / f_\pi^2 = 2\rho \Sigma_A^{(\pi)} / f_\pi^2 m_\pi^2$. This pion cloud contribution to the nuclear sigma term has already been obtained in [19] with a different method. Here we derive its explicit form using the analytical structure of the pion propagator in eq.14. Let us introduce the full longitudinal spin-isospin response function related to the imaginary part of the full pion polarization propagator Π_L according to :

$$
\begin{aligned}
R_L(\omega, \mathbf{k}) &= -\frac{V}{\pi} Im\, \Pi_L(\omega, \mathbf{k}) = -\frac{V}{\pi} Im \left(\mathbf{k}^2 \tilde{\Pi}_0(\omega, \mathbf{k}) \, \frac{\omega^2 - \omega_k^2}{\omega^2 - \omega_k^2 - \mathbf{k}^2 \tilde{\Pi}_0(\omega, \mathbf{k})} \right) \\
&= 3 \left(\frac{g_{\pi NN}}{2M_N} \right)^2 v^2(\mathbf{k}) \sum_n \left| < n| \sum_{i=1}^{A} \vec{\sigma}(i).\mathbf{k}\, \tau_\alpha(i)\, e^{i\mathbf{k}.\mathbf{x}(i)} |0> \right|^2 \delta(E_n - \omega) \qquad (16)
\end{aligned}
$$

where $g_{\pi NN} = M_N g_A / f_\pi$ and $v(\mathbf{k})$ are the πNN coupling constant and form factor. Once the vacuum contribution is substracted $< \Phi^2 >$ can be calculated using a dispersion relation for Π_L :

$$
\begin{aligned}
< \Phi^2 > &= 3 \int \frac{i\, d^4k}{(2\pi)^4} \Big(D_R(k) - D_0(k) \Big) = 3 \int \frac{i\, d^4k}{(2\pi)^4} \left(\frac{1}{k_0^2 - \omega_k^2 + i\eta} \right)^2 \Pi_L(k) \\
&= 3 \int \frac{d^3k}{(2\pi)^3} \int_{-\infty}^{+\infty} \frac{i dk_0}{2\pi} \left(\frac{1}{k_0^2 - \omega_k^2 + i\eta} \right)^2 \int_0^{+\infty} d\omega \left(\frac{-2\omega}{\pi} \right) \frac{Im \Pi_L(\omega, \mathbf{k})}{k_0^2 - \omega^2 + i\eta} \qquad (17)
\end{aligned}
$$

Exchanging the ordering of the k_0 and ω integrations and performing a standard contour k_0 integration, we finally obtain for $\Sigma_A^{(\pi)}$:

$$
\Sigma_A^{(\pi)} = \frac{3m_\pi^2}{2A} \int \frac{d^3k}{(2\pi)^3} \int_0^\infty d\omega \left(\frac{1}{2\omega_k^2(\omega + \omega_k)^2} + \frac{1}{2\omega_k^3(\omega + \omega_k)} \right) R_L(\omega, \mathbf{k}) \qquad (18)
$$

Taking the zero density limit, we recover the pion cloud contribution to the pion-nucleon sigma term whose explicit form (ignoring the delta width) is :

$$
\begin{aligned}
\Sigma_N^{(\pi)} &= \frac{3m_\pi^2}{2} \int \frac{d^3k}{(2\pi)^3} \left(\frac{g_{\pi NN}}{2M_N} \right)^2 v^2(\mathbf{k}) \left[\left(\frac{1}{2\omega_k^2(\epsilon_k + \omega_k)^2} + \frac{1}{2\omega_k^3(\epsilon_k + \omega_k)} \right) \right. \\
&+ \left. \frac{16}{9} \left(\frac{g_{\pi NN}}{g_{\pi N\Delta}} \right)^2 \left(\frac{1}{2\omega_k^2(\omega_{\Delta k} + \omega_k)^2} + \frac{1}{2\omega_k^3(\omega_{\Delta k} + \omega_k)} \right) \right] \qquad (19)
\end{aligned}
$$

with $\epsilon_k = k^2/2M_N$ and $\omega_{\Delta k} = M_\Delta - M_N + k^2/2M_\Delta$. $\Sigma_N^{(\pi)}$ has already been evaluated within chiral quark models like CBM [21] providing a form factor from the nucleon size and specific value for $g_{\pi NN}/g_{\pi N\Delta}$. It has been found of the order of 30 MeV with about 10 MeV from the delta sector, the rest (15 MeV) coming from the scalar quark density inside the nucleon. Now, the interesting

question is how much the π-nucleus sigma term per nucleon deviates from the π-nucleon one or, said differently , what is the pion exchange contribution ? This problem has already been adressed in [19]; let us briefly summarize the conclusion of this work. The nucleon-hole sector contribution can be reasonnably calculated within a static approximation, i.e. putting $\omega = 0$ in the prefactor of eq.(18); the energy integration yields a two-body spin-isospin ground state matrix element once the free nucleonic part has been substracted. According to a calculation with correlated wave functions, it turns out that the Pauli blocking effect is almost exactly compensated by the effect of tensor correlations. Hence, the nucleonic sector itself does not contribute to the modification of the sigma term. For the delta sector, the situation is less clear. The occurence of collective pion-delta states (the so-called pionic branch) shifts part of the strength at lower energy and one can expect an increase of the sigma commutator closely related to the pion excess in nuclei. This expected feature is consistent with a dispersive analysis a la Fubini-Furlan which gives an increase of the sigma term of about 5 MeV. However the full calculation of eq.(18) with a realistic longitudinal spin-isospin response function, including $p-h$, $\Delta - h$ and $2p-2h$ sectors remains to be done [24].

4 Pion propagator and in-medium pion-nucleon coupling constant

In addition to the usual p-wave piece, the pion self-energy receives a contribution of the pion loop which contains the effect of the modification of the πNN vertex present in the pion-pion quasi-potential. It reads :

$$S(\omega, \mathbf{q}) = \mathbf{q}^2 \tilde{\Pi}_0(\omega, \mathbf{q}) \, + \, \frac{<\Phi^2>}{6 f_\pi^2} \left[m_\pi^2 - 2\beta \left(\omega^2 - \omega_q^2 - \mathbf{q}^2 \tilde{\Pi}_0(\omega, \mathbf{q}) \right) \right] \tag{20}$$

The full inverse pion propagator with pion loop effect takes the form :

$$\begin{aligned} \tilde{D}^{-1}(\omega, \mathbf{q}) \; &= \; \omega^2 - \omega_q^2 \, - \, S(\omega, \mathbf{q}) \\ &= \; \left(1 + \beta \frac{<\Phi^2>}{3 f_\pi^2} \right) \left(\omega^2 - \omega_q^2 - \mathbf{q}^2 \tilde{\Pi}_0(\omega, \mathbf{q}) \right) \, + \, \frac{<\Phi^2>}{6 f_\pi^2} \, m_\pi^2 \end{aligned} \tag{21}$$

As expected from the effective πNN effective lagrangian of eq. (7), the p-wave self enegy is modified, in this one-pion loop approximation by a factor $1 + \beta <\Phi^2> /3 f_\pi^2 \approx (1 + \beta <\Phi^2> /6 f_\pi^2)^2$. However, as we will see just below, a wave-function renormalization will just compensate this effect. To clearly see what is going on, we drop the pure s-wave part of the self-enegy and concentrate on the p-wave part. In this case the full pion propagator becomes :

$$\tilde{D}(\omega, \mathbf{q}) = \gamma \, D_R(\omega, \mathbf{q}) = \frac{\left(1 + \beta \frac{<\Phi^2>}{3 f_\pi^2} \right)^{-1}}{\omega^2 - \omega_q^2 - \mathbf{q}^2 \tilde{\Pi}_0(\omega, \mathbf{q})} \tag{22}$$

The effective πNN coupling constant is modified by the vertex correction of (7) and the wave-function renormalization factor $\sqrt{\gamma}$ which compensate each other to one pion loop order, namely :

$$\frac{g^*_{\pi NN}(\rho)}{g_{\pi NN}} = \sqrt{\gamma} \, \left(1 + \beta \frac{<\Phi^2>}{6 f_\pi^2} \right) = \, 1 \, + \, \mathcal{O}(<\Phi^4>) \tag{23}$$

Hence, to one-pion loop, the pion-nucleon coupling constant is unmodified in the nuclear medium. Similarly, if we consider pion exchange between two nucleons the two vertex renormalizations

exactly compensate the γ factor of the propagator. Consequently, the effective pion propagator, whose imaginary part is directly related to the full pionic response function, is just the usual D_R calculated with an irreductible pion self-energy Π_0 which is also unmodified by pion loop effects. This result justifies a posteriori all the phenomenological studies of pion-nucleus interaction [25] and charge exchange reaction [1, 2, 3]. The occurence of this γ factor has been previously established in [26] at finite temperature for a hot pion gas for which $<\Phi^2>= T^2/4$ in the chiral limit. Our result is also valid in this case and one can conclude that the πNN coupling constant is also unmodified to order T^2 as previously stated in [27].

In recent papers [20, 26], we established, within the same chiral lagrangian, that the axial coupling constant g_A and the pion decay constant f_π are in-medium modified according to :

$$\frac{f_\pi^*}{f_\pi} = \frac{g_A^*}{g_A} = 1 - \frac{<\Phi^2>}{3f_\pi^2} \tag{24}$$

At finite temperature, in a hot pion gas, this quenching effect acting on the axial current is accompanied by the mixing of the vector and axial correlators as demonstrated in [28]. Since $g_{\pi NN}$ [27] and the nucleon mass [29] are unmodified, we come to the conclusion that the Goldberger-Treiman relation $M_N\, g_A = g_{\pi NN}\, f_\pi$ is preserved at finite temperature, as already pointed out in [27]. At finite density the quenching of g_A and f_π is also linked to a mixing effect intimately related to chiral symmetry restoration [20]. The novel feature is that the unmodified Goldberger-Treiman relation also holds at finite density in the nuclear medium since there is no modification of the nucleon mass apart higher order effects associated to the p-wave dressing of the nucleonic pion cloud. However we stress that this result is strictly valid with neglect of short-range correlations affecting the nucleon observables g_A and $g_{\pi NN}$. In [20] we have shown that the renormalization of g_A (which is a pure vertex renormalization) is accounted for by simply replacing the averaged pion scalar density $<\Phi^2>$ by an effective one incorporating the effect of short-range correlation; the net result is a 10% quenching at normal nuclear density. The problem of the effect of short-range correlation on $g_{\pi NN}$ remains to be clarified.

5 Pion-pion scattering in the nuclear medium

We are now in position to study pion-pion scattering both in free space and in the nuclear medium. In particular, we want to investigate to which extent the medium modification of the $\pi\pi$ interaction, involving the pion polarizability Π_0 (eq.(12)), influences the two-pion strength function.

Let us first consider the free case; the unitarized scattering amplitude in the scalar-isoscalar channel can be obtained as the solution of a Lippmann-Schwinger equation :

$$< q_a q_b|\mathcal{T}_0|q_c q_d >=< q_a q_b|\mathcal{M}_0|q_c q_d > + \frac{1}{2}\int \frac{id^4k_1}{(2\pi)^4} < q_a q_b|\mathcal{M}_0|k_1 k_2 > D_0(k_1)D_0(k_2) < k_1 k_2|\mathcal{T}|q_c q_d > \tag{25}$$

where $k_2 = P - k_1 = q_a + q_b - k_1$ and D_0 is the free space pion propagator. The potential $\mathcal{M}_0$ given by eq.(10) contains, together with the on-shell piece, an off-shell piece depending on the four inverse pion propagators. However, as explained in [18], the off-shell terms can always be absorbed in the potentiel by a proper redefinition of the coupling constants and mass. It follows that the free scattering matrix can be algebraically obtained as :

$$\mathcal{T}(s) = \frac{1}{f_\pi^2}(m_\pi^2 - 2s)\left(1 - \frac{1}{2f_\pi^2}(m_\pi^2 - 2s)\int \frac{d^3k}{(2\pi)^3}\frac{v^2(k)}{\omega_k}\frac{1}{s - 4\omega_k^2 + i\eta}\right)^{-1} \tag{26}$$

where $v(k)$ is a form factor which can be fitted on low energy phase shifts and scattering length. Evidently, an explicit sigma meson and/or a coupling to the $K\bar{K}$ channel should be introduced to describe the higher energy part [14, 18] but this is not the purpose of this paper. The question of interest here is to study the threshold region of this pion-pion amplitude in the nuclear medium. According to the approach of [14], this in-medium scattering matrix is obtained by simply replacing in (26) the free pion propagators by the dressed one containing the p-wave pion polarizability. However, as pointed out in [18], one has to take care of the off-shell piece of the effective in-medium potential (12) which contains a piece proportionnal to the p-wave polarizability. As claimed in [18], this yields a contribution to the scattering amplitude, having in the intermediate state one (dressed) pion and one (undressed) particle-hole or delta-hole state (see fig.2c). The effect of this $\pi - ph$ configuration (calculated in a particular representation $\alpha = 0$) was considered in [18] yielding an accumulation of strength below threshold at moderate density ($\rho/\rho_0 = 0.5$); the unusual feature was the disparition of this effect with increasing density. What is also surprising is that this effect comes from a contribution to the in-medium potential (12) which is manifestly representation dependent. To clarify the matter we will study the first order contribution to the scattering amplitude ($V^{eff} \,.\, 2\pi\, propagator \,.\, V^{eff}$) to first order in the polarization propagator Π_0; the various diagrams depicted in fig. 2 are the same considered in [18]. As in this paper we consider only the pieces having an imaginary part since we are mainly interested in the 2π strength function. The first diagram (fig.2a) is the usual one and corresponds to the dressing of the pion but with the potential corrected by the off-shell behaviour (eq.(10)) :

$$\mathcal{T}^{(a)} = 2 \times \frac{1}{2} \int \frac{i\, d^4k_1}{(2\pi)^4} \left[V_{on} - \frac{\beta}{f_\pi^2} D_0^{-1}(k_2) \right]^2 \mathbf{k}_2^2 \tilde{\Pi}_0(k_2) D_0^2(k_2)\, D_0(k_1) \tag{27}$$

The second diagram (fig.2b) explicitely incorporates the 4π vertex correction involving the polarizability $\tilde{\Pi}_0$

$$\mathcal{T}^{(b)} = 4 \times \frac{1}{2} \int \frac{i\, d^4k_1}{(2\pi)^4} \left[V_{on} - \frac{\beta}{f_\pi^2} D_0^{-1}(k_2) \right] \frac{\beta}{f_\pi^2} \mathbf{k}_2^2 \tilde{\Pi}_0(k_2) D_0^2(k_2)\, D_0(k_1) \tag{28}$$

Finally, there is a diagram (fig.2c) with a $p - h$ bubble in the intermediate state. It gives a contribution (at zero total momentum, $\mathbf{P} = 0$)

$$\mathcal{T}^{(c)} = 2 \times \frac{1}{2} \int \frac{i d^4k_1}{(2\pi)^4} \left(\frac{\beta}{f_\pi^2} \right)^2 \mathbf{k}_2^2 \tilde{\Pi}^0(k_2)\, D_0(k_1) \tag{29}$$

The various prefactors come from the counting of equivalent diagrams. Summing this three contributions, we see that the representation dependent piece (*i.e.* involving β) disappears as it should be.

$$\mathcal{T}^{(a)} + \mathcal{T}^{(b)} + \mathcal{T}^{(c)} = \int \frac{i d^4k_1}{(2\pi)^4} V_{on}^2\, \mathbf{k}_2^2\, \tilde{\Pi}_0(k_2)\, D_0^2(k_2)\, D_0(k_1) \tag{30}$$

Hence we find the "normal" first order dressing of the pion propagator. This result is at variance with [18] where the $\pi - ph$ intermediate state was found to contribute to the strength function. This result can be generalized when the pion propagator is fully dressed by the p-wave pion polarizability. The total second order scattering amplitude takes the form (for $\mathbf{P} = 0$) :

$$Im\mathcal{T}^{(2)} \;=\; \frac{1}{2} Im \int \frac{i d^4k_1}{(2\pi)^4} \left\{ \left[V_{on} - \frac{\beta}{f_\pi^2} \left(D_R^{-1}(k_1) + D_R^{-1}(k_2) \right) \right]^2 D_R(k_1)\, D_R(k_2) \right.$$

$$
\begin{aligned}
& + \left(\frac{\beta}{f_\pi^2}\right)^2 \left(\mathbf{k}_1^2\,\tilde{\Pi}_0(k_1)\,D_R(k_2)\,+\,\mathbf{k}_2^2\,\tilde{\Pi}_0(k_2)\,D_R(k_1)\right)\Big\} \\
= \; & \frac{1}{2}\,Im\int\frac{id^4k_1}{(2\pi)^4}V_{on}^2\,D_R(k_1)\,D_R(k_2) \qquad (31)
\end{aligned}
$$

Although some more work remains to be done to write a fully unitarized in-medium scattering equation, we come to the conclusion that intermediate states with one quasi-pion and undressed $p-h$ or $\Delta-h$ states do not contribute to the stength function. This is not so surprising since only fully dressed states (*i e.* exact eigenstates) can appear in the intermediate state of the $\pi\pi$ scattering series.

6 Conclusion

Based on a chiral lagrangian we have investigated various effects concerning pion interaction in the nuclear medium. We have in particular studied the interplay between p-wave interaction responsible for the existence of collective pionic modes and s-wave πN interaction. We have insisted on the key role played by the in-medium scalar pion density which governs a large amount of chiral symmetry restoration. Starting from an in-medium modified $\pi\pi$ interaction consistent with the p-wave dressing of the pion, we have obtained an explicit expression of the pion scalar density and the pion cloud contribution to the π-nucleus sigma term. We have shown that, contrary to the axial observables f_π and g_A, the πNN coupling constant remains unmodified to one-pion loop order. As a consequence the Goldberger-Treiman relation is preserved at finite density, at least in the limit where the effect of short-range correlations is ignored. We have finally demonstrated that the in-medium modification of the $\pi\pi$ potential cannot give genuine contribution to the two-pion strength function, confirming the validity of the approach developped in [14]. This is particularly important in view of the understanding of the structure observed in the two-pion invariant mass spectrum in $A(\pi,2\pi)$ reactions [30].

Acknowledgements : We thank J. Delorme and M. Ericson for enlightening discussions and critical reading of the manuscript.

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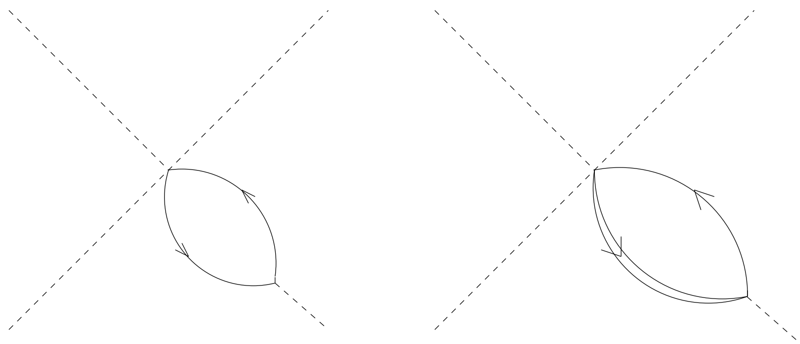
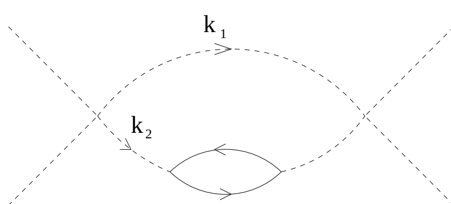
FIGURE CAPTIONS

Figure 1 : Vertex correction to the $\pi\pi$ interaction in the medium through $p-h$ and $\Delta-h$ polarization bubbles which may contain the screening effect due to short range correlation (g' parameters).

Figure 2 : Medium effects appearing in $\pi\pi$ rescattering. 2a : dressing of the intermediate pion by the p-wave polarizability. 2b : vertex correction to the $\pi\pi$ potential. 2c : contribution of the $p-h$ and $\Delta-h$ excitations in the intermediate state.



2a



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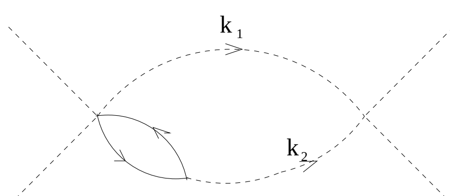

2c